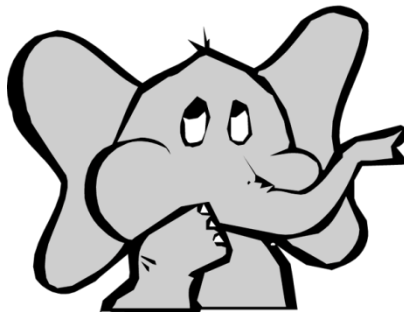
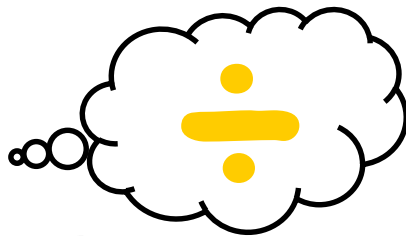
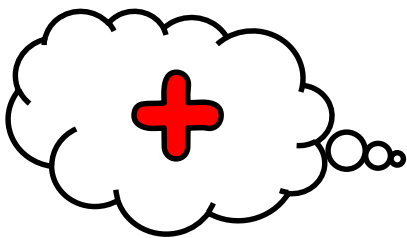
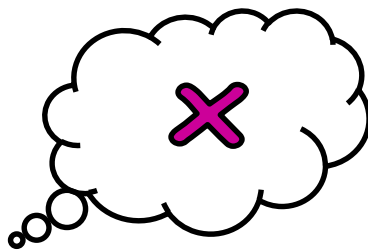
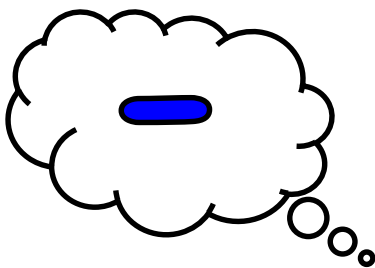


# Progression in Calculations



## Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept.

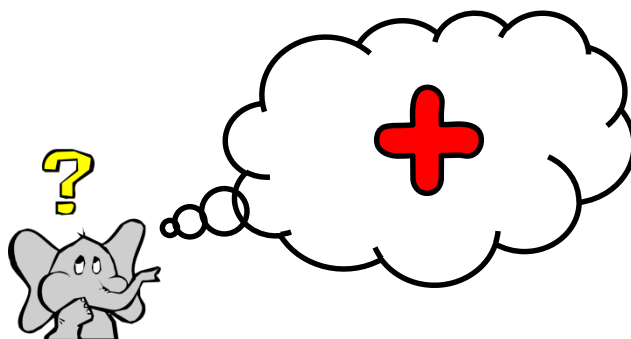
The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

## Progression in Teaching Addition

### Mental Skills

Recognise the size and position of numbers  
Count on in ones and tens  
Know number bonds to 10 and 20  
Add multiples of 10 to any number  
Partition and recombine numbers  
Bridge through 10



### Models and Images

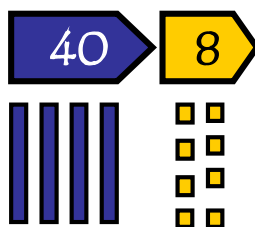
Counting apparatus  
Place value apparatus  
Place value cards  
Number tracks  
Numbered number lines  
Marked but unnumbered number lines  
Empty number lines  
Hundred square

Counting stick

Bead string

Models and Images charts

ITPs - Number Facts, Ordering Numbers, Number Grid, Counting on and back in ones and tens



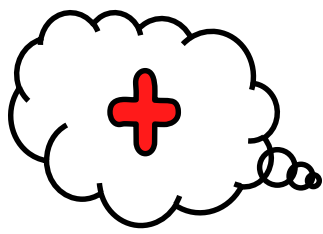
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



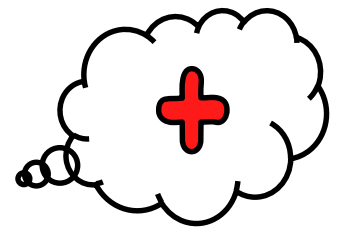
### Key Vocabulary

add  
addition  
plus  
and  
count on  
more  
sum  
total  
altogether  
increase

add and count on  
addition plus  
more sum total  
altogether increase



# Progression in written methods for Addition



**Number Track**



**Number Line**



**Expanded method  
Partitioning and recombining**

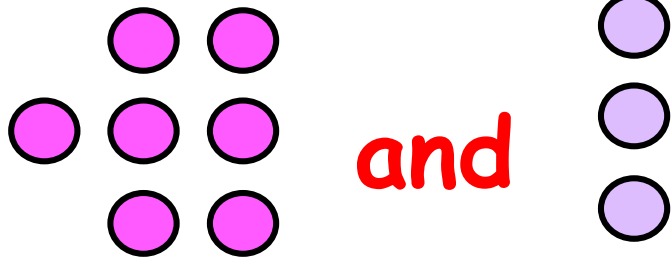


**Compact Method**

# Stage 1 - Number Track (Year 1)



$$7 + 3$$



- understand addition is combining groups of objects
- count on using a number track

$$\begin{array}{c} \textcircled{2} + \textcircled{1} = \textcircled{3} \\ \text{🐻 🐻} \quad \text{🐻} \quad \text{🐻 🐻 🐻} \end{array} \quad \begin{array}{c} \textcircled{1} + \textcircled{2} = \textcircled{3} \\ \text{🐻} \quad \text{🐻 🐻} \quad \text{🐻 🐻 🐻} \end{array}$$

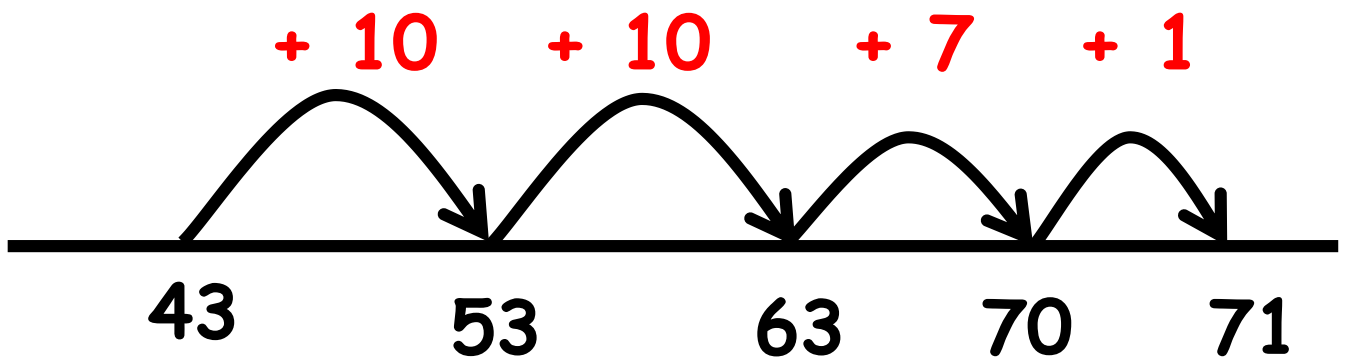
- understand addition can be done in any order

## Stage 2 - Using a number line

(numbered, partially numbered, blank)

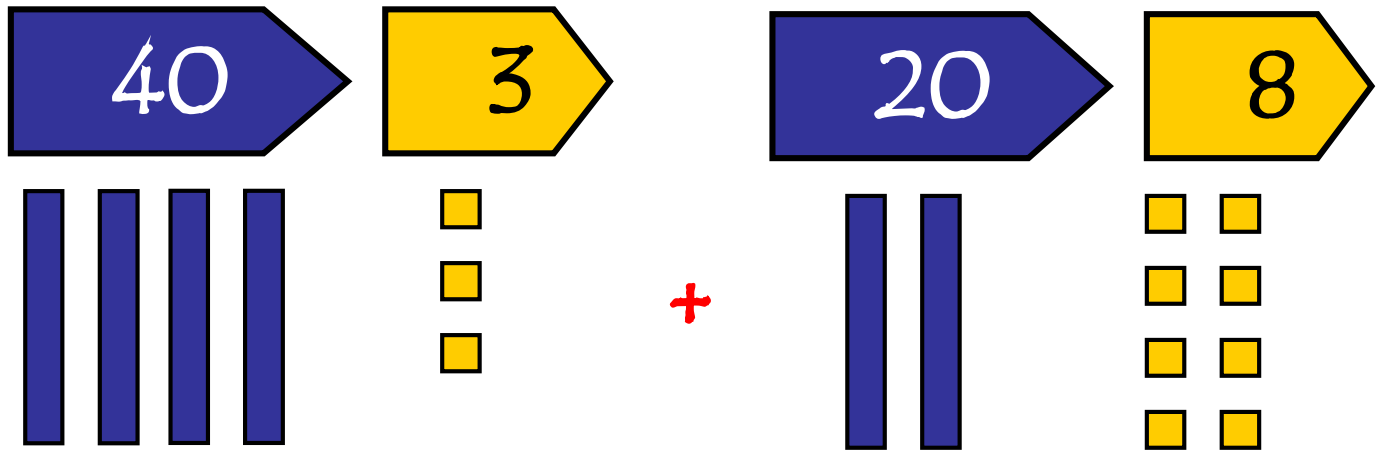
Years 2/3

$$43 + 28$$



- ESTIMATE first
- teach and encourage children to partition numbers in different ways
- start from the largest number
- encourage use of number line for jottings

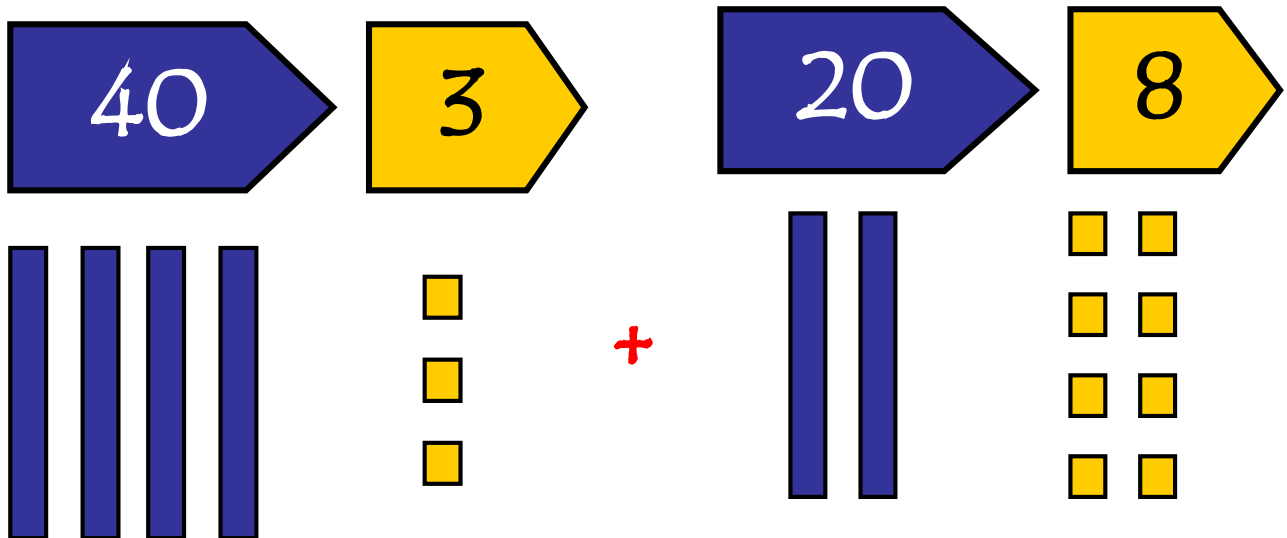
# Stage 3 - Partitioning & Recombining The Expanded Method (Year 3)



40	+	20	=	60
3	+	8	=	11
<hr/>				
60	+	11	=	71

- Encourage ESTIMATION
- use place value cards to partition
- use place value apparatus (dienes) to reinforce 'carrying'
- use equipment alongside expanded written method to bridge from concrete to abstract

# Stage 4 - Expanded Method leading to Compact Method (Year 4 onwards)



4	0	+	3		
2	0	+	8		
6	0	+	1	1	= 7 1

4	3
+	<u>2 8</u>
7	<u>1</u>
1	

Expanded method leading to compact method



## Progression in Teaching Subtraction

### Mental Skills

Recognise the size and position of numbers

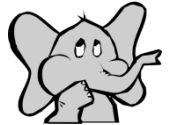
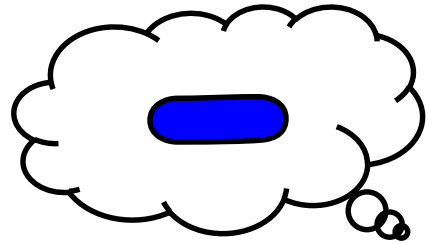
Count back in ones and tens

Know number facts for all numbers to 20

Subtract multiples of 10 from any number

Partition and recombine numbers (only partition the number to be subtracted)

Bridge through 10



### Models and Images

Counting apparatus

Place value apparatus

Place value cards

Number tracks

Numbered number lines

Marked but unnumbered lines

Hundred square

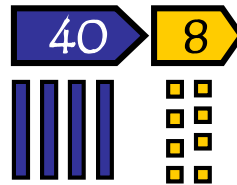
Empty number lines.

Counting stick

Bead strings

Models and Images Charts

ITPs - Number Facts, Counting on and back in ones and tens, Difference



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



### Key Vocabulary

subtract

take away

minus

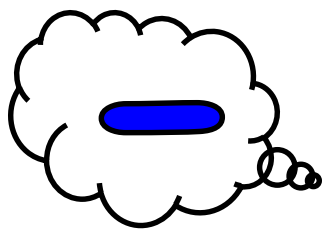
count back

less

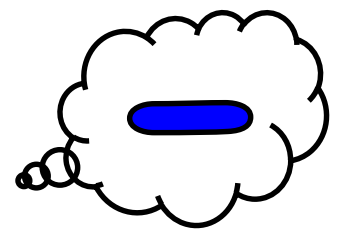
fewer

difference between

count back   take away  
fewer   subtract  
minus   less  
difference between



Progression in  
written methods  
for Subtraction



**Number Track**



**Number Line**  
(Finding the difference  
and counting back)

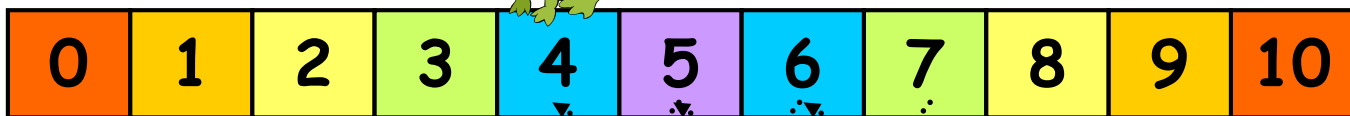


**Expanded method**  
**Partitioning and recombining**



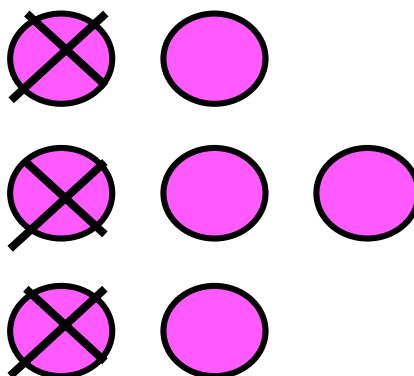
**Compact Method**

# Stage 1 - Number Track Year 1



$$7 - 3$$

seven take away three

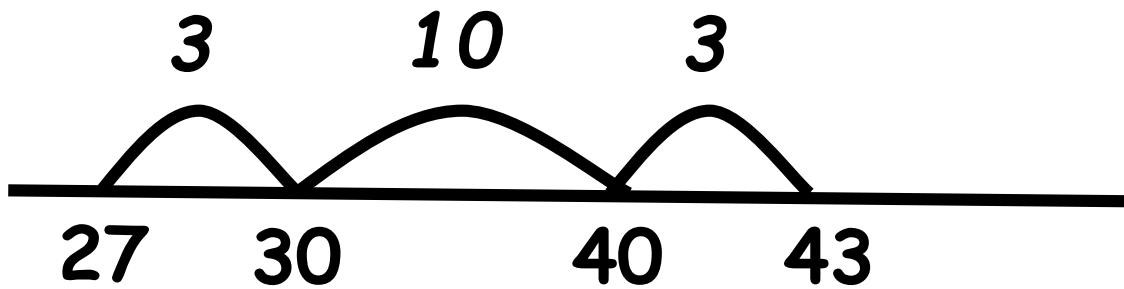


- understand subtraction is taking away objects
- count back along a number track

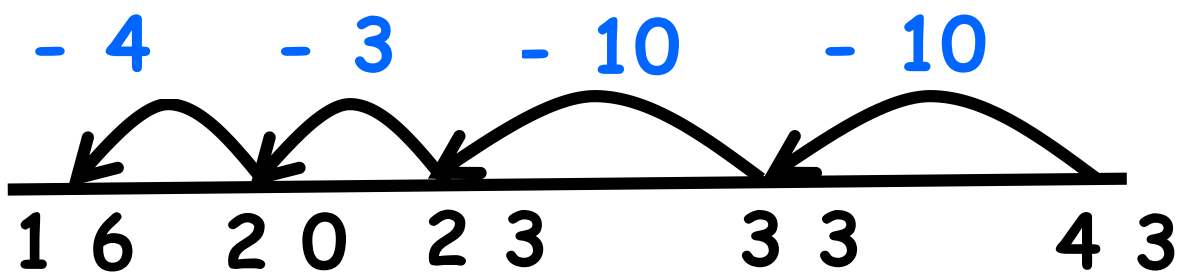
# Stage 1 - Using a number line (Years 2 and 3)

$$43 - 27$$

finding the difference



counting back



- ESTIMATE first
- encourage children to partition numbers in different ways
- understand 'finding the difference' AND 'counting back' has the same result

# Stage 2 - Partitioning & Recombining Exchanging using apparatus

(Year 3)

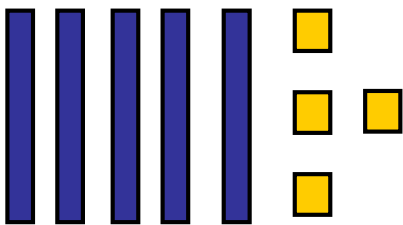
$$54 - 27$$

54

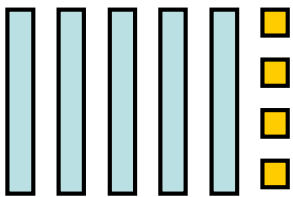
-

20

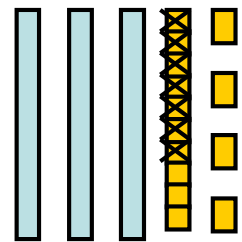
7



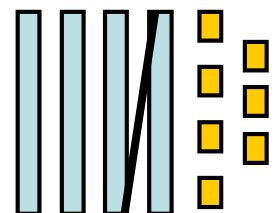
$$54 - 27 =$$



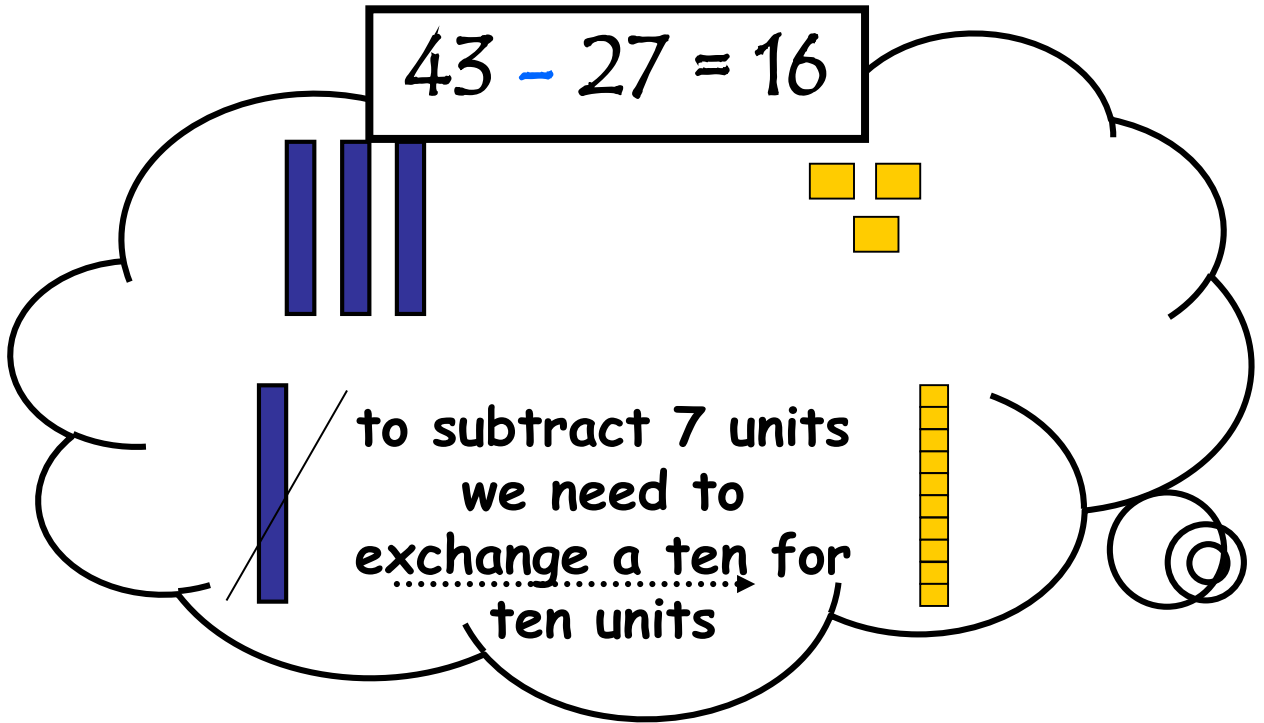
$$54 - 7 = 47$$



$$47 - 20 = 27$$



# Stage 3 - Expanded Method (Year 4)



$$\begin{array}{r} 30 \\ \cancel{40} \\ - 20 \\ \hline 10 \end{array} \quad \begin{array}{r} 10 + 3 \\ 7 \\ \hline 6 \end{array}$$

10 and 6

- move from concrete apparatus to expanded written method
- continue to encourage ESTIMATION

## Stage 4 - Compact Method (Year 5 onwards)

$$\begin{array}{r} \overset{30}{\cancel{40}} \quad \quad \quad 10 + 3 \\ - \quad 20 \quad \quad \quad 7 \\ \hline 10 \quad \text{and} \quad 6 \end{array}$$

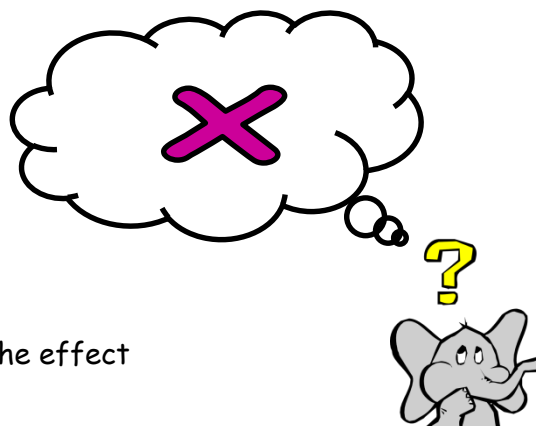
$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{1}{3} \\ - \quad 27 \\ \hline 16 \end{array}$$

- move from expanded written method to compact method
- continue to encourage ESTIMATION

## Progression in Teaching Multiplication

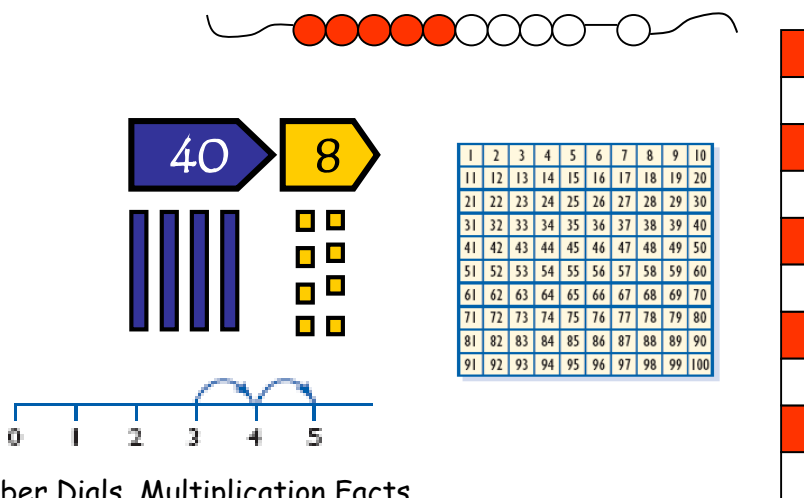
### Mental Skills

Recognise the size and position of numbers  
 Count on in different steps 2s, 5s, 10s  
 Double numbers up to 10  
 Recognise multiplication as repeated addition  
 Quick recall of multiplication facts  
 Use known facts to derive associated facts  
 Multiplying by 10, 100, 1000 and understanding the effect  
 Multiplying by multiples of 10



### Models and Images

Counting apparatus  
 Place value apparatus  
 Arrays  
 100 squares  
 Number tracks  
 Numbered number lines  
 Marked but unnumbered lines  
 Empty number lines.  
 Multiplication squares  
 Counting stick  
 Bead strings  
 Models and Images charts  
 ITPs - Multiplication grid, Number Dials, Multiplication Facts

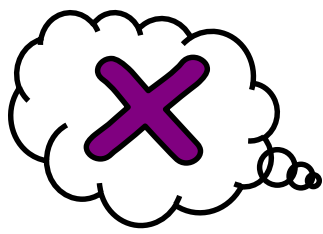


### Vocabulary

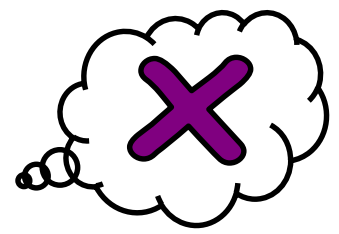
lots of  
 groups of  
 times  
 multiply  
 multiplication  
 multiple  
 product  
 once, twice, three times  
 array, row, column  
 double  
 repeated addition

**multiplication**      **product**  
**once, twice, three times**  
**double**      **groups of**  
**repeated addition**      **lots of**  
**array, row, column**      **multiply**  
**times**      **multiple**





Progression in  
written methods  
for multiplication



Repeated addition, arrays



Grid method  
(with imagery)

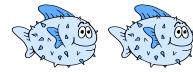
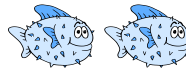
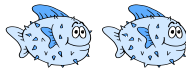


Grid method



Long multiplication

# Stage 1 - Repeated addition, arrays (Year 2)



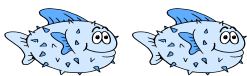
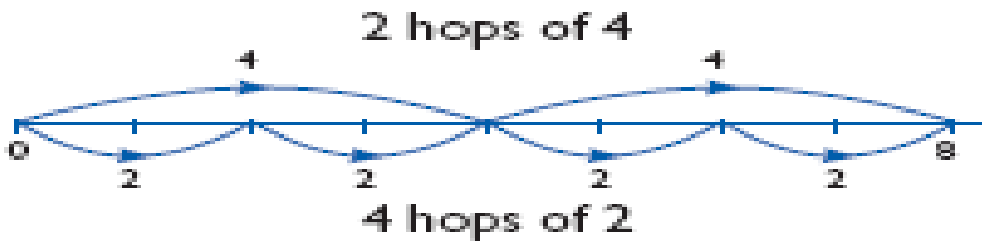
$$2 + 2 + 2 + 2$$

$$2 + 2 + 2 + 2 = 8$$

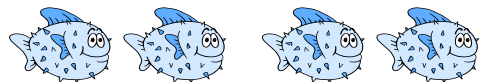
$$4 \times 2 = 8$$

2 multiplied by 4

4 lots of 2



$$2 \times 4$$

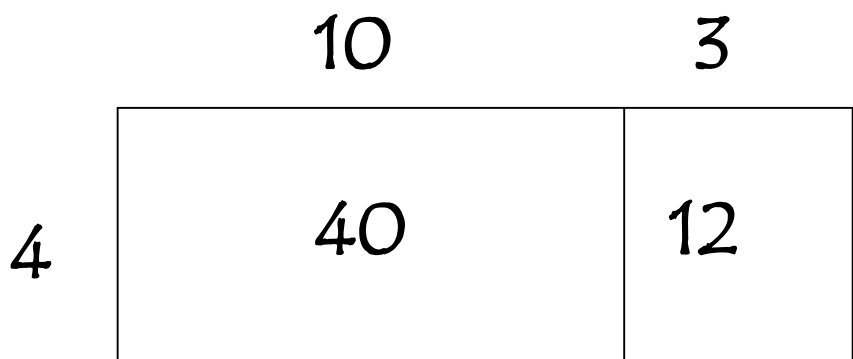
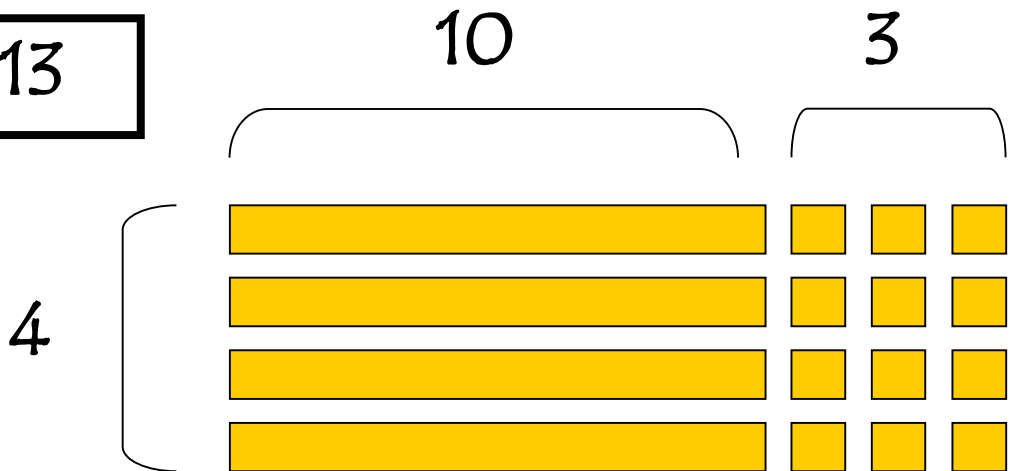


$$4 \times 2$$

- understand that multiplication is a shortened form of repeated addition
- understand multiplication as arrays and jumps on a number line

## Stage 2 - Modelling grid method with place value equipment (Year 3)

$$4 \times 13$$

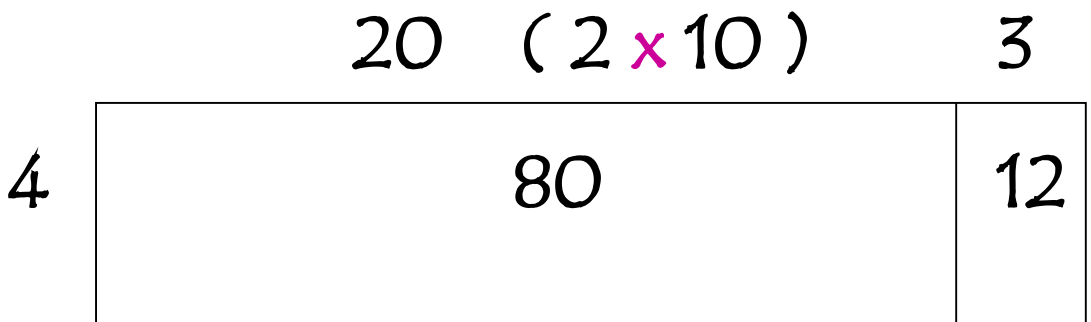
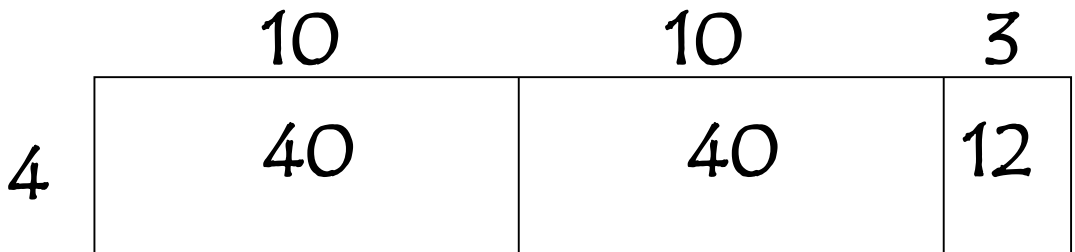
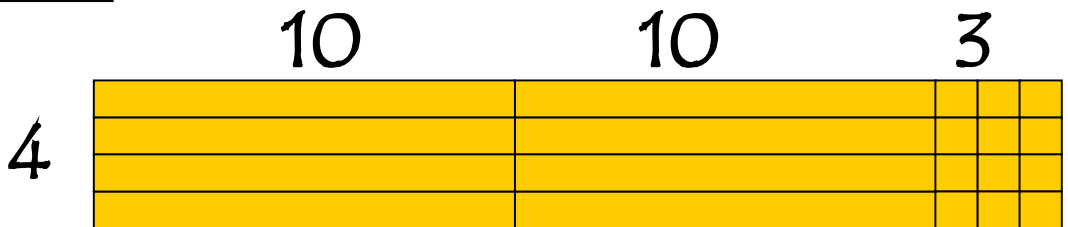


$$40 + 12 = 52$$

- use place value equipment to illustrate grid method
- do not move above  $\times 19$  until understanding of method is secure

# Stage 3 - Modelling grid method with place value equipment (multiples of 10) (Year 4)

$$4 \times 23$$



$$80 + 12 = 92$$

- use place value equipment to illustrate grid method with multiples of 10

## Stage 4 - Long multiplication (Years 5 and 6)

$$\begin{array}{r} \overset{4}{\cancel{1}} \quad 56 \\ \times \quad 27 \\ \hline 1120 \\ \quad 392 \\ \hline 1512 \\ \quad 1 \end{array}$$

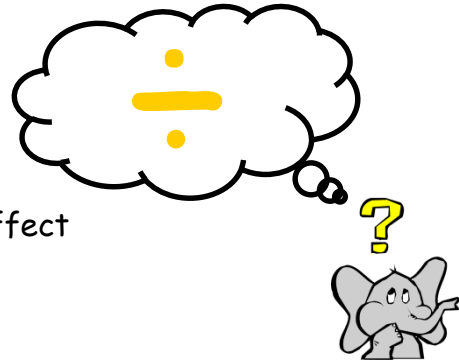
$(56 \times 20)$   
 $(56 \times 7)$

- ONLY move on to this method if understanding is secure

# Progression in Teaching Division

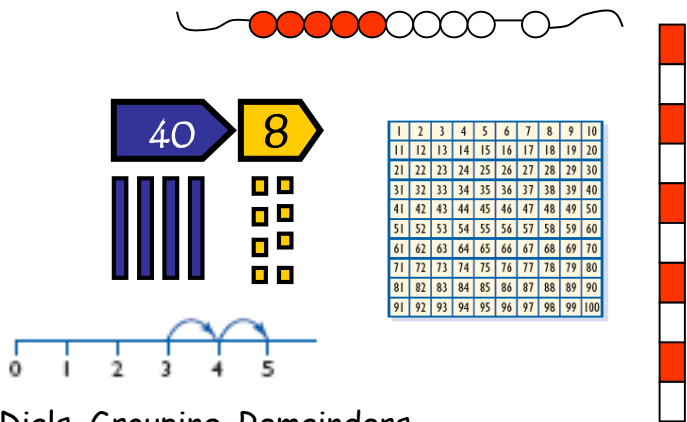
## Mental Skills

- Recognise the size and position of numbers
- Count back in different steps 2s, 5s, 10s
- Halve numbers to 20
- Recognise division as repeated subtraction
- Quick recall of division facts
- Use known facts to derive associated facts
- Divide by 10, 100, 1000 and understanding the effect
- Divide by multiples of 10



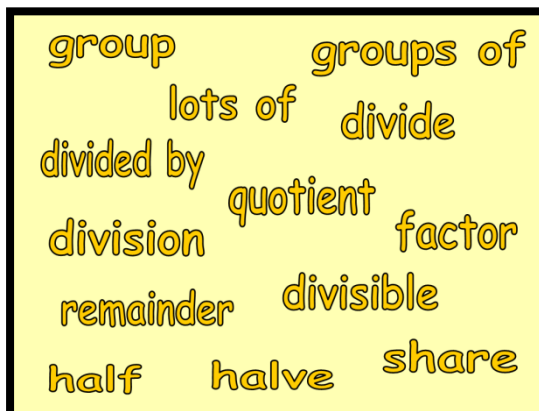
## Models and Images

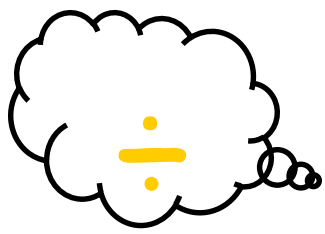
- Counting apparatus
- Arrays
- 100 squares
- Number tracks
- Numbered number lines
- Marked but unnumbered lines
- Empty number lines.
- Multiplication squares
- Models and Images charts
- ITPs - Multiplication grid, Number Dials, Grouping, Remainders



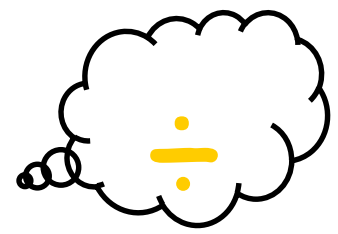
## Vocabulary

- lots of
- groups of
- share
- group
- halve
- half
- divide
- division
- divided by
- remainder
- factor
- quotient
- divisible





# Progression in written methods for division



Division as sharing and grouping



Grouping on a number line



Vertical recording

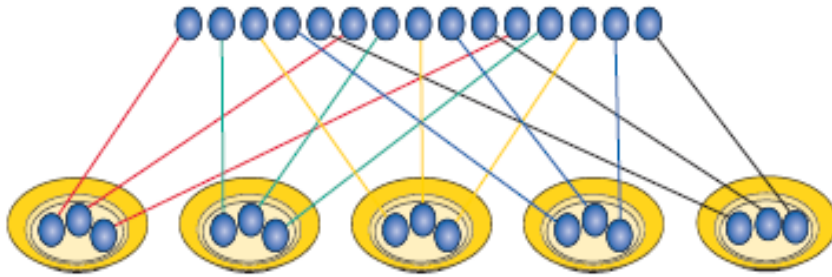


Chunking (fact box)

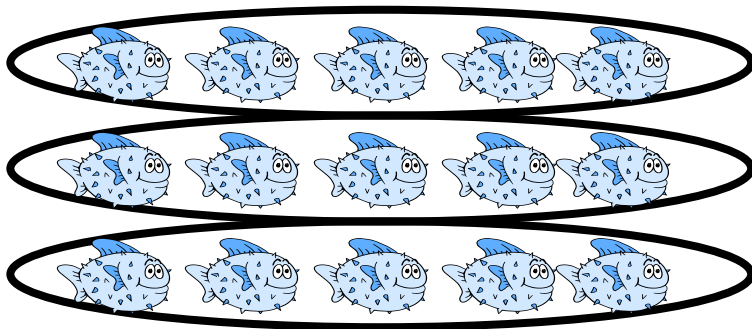


Short/long division

# Stage 1 - Division as sharing and grouping (year 2)

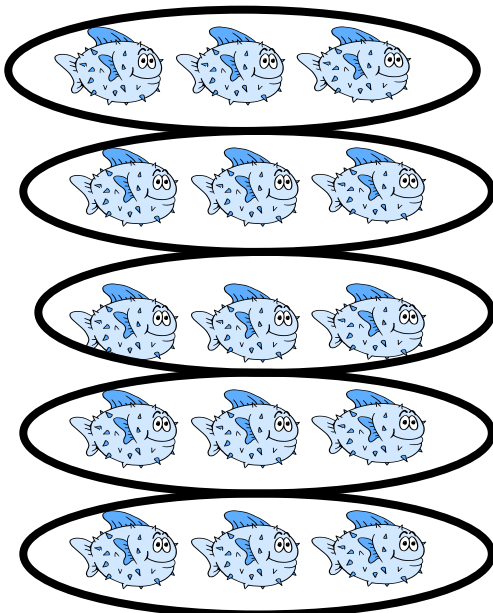


*sharing  
one at  
a time*



$$15 \div 3$$

*15 divided into  
3 equal groups*



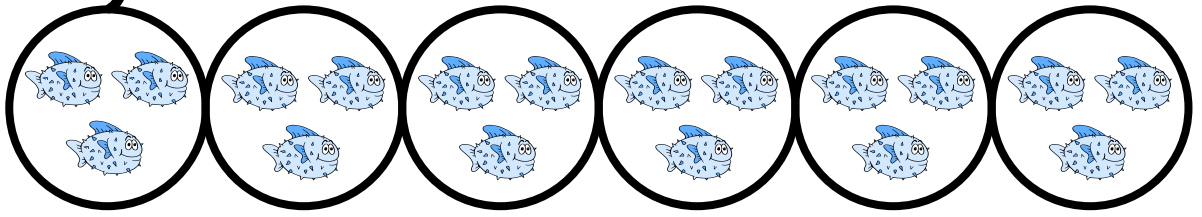
$$15 \div 5$$

*15 divided into  
5 equal groups*

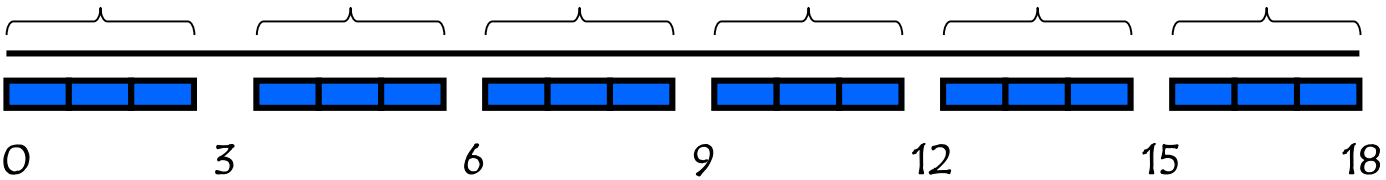
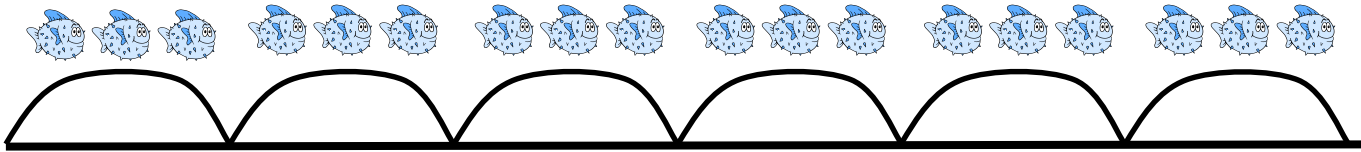
- understand division as sharing, understand division as grouping
- understand remainders



# Stage 2 - Grouping on a number line (Year 3)

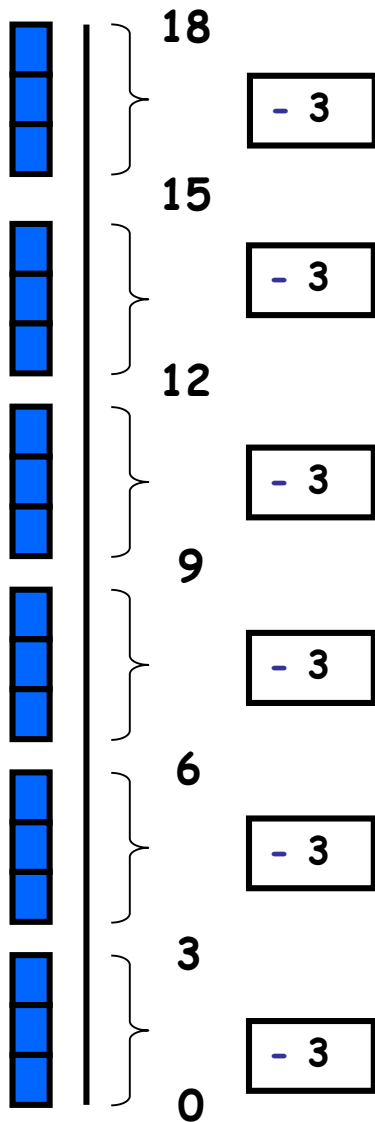
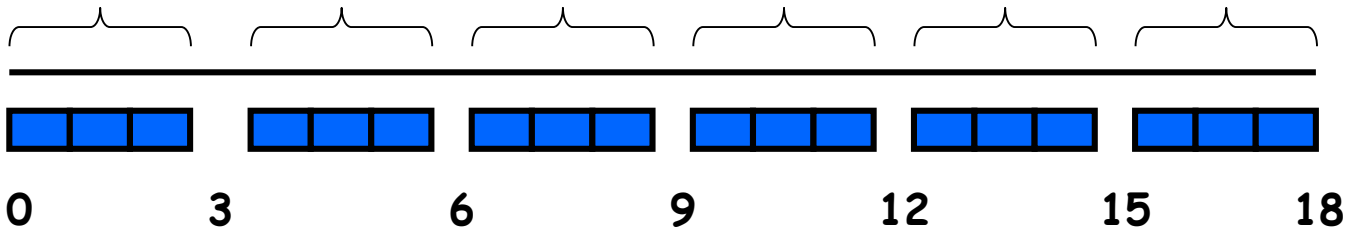


$$18 \text{ --- } 3 \text{ --- } 3 \text{ --- } 3 \text{ --- } 3 \text{ --- } 3 \text{ --- } 3$$



- understand that division is repeated subtraction
- understand division as groups on a number line

# Stage 3 - Vertical recording (Year 4)



$$18 \div 3 = 6$$

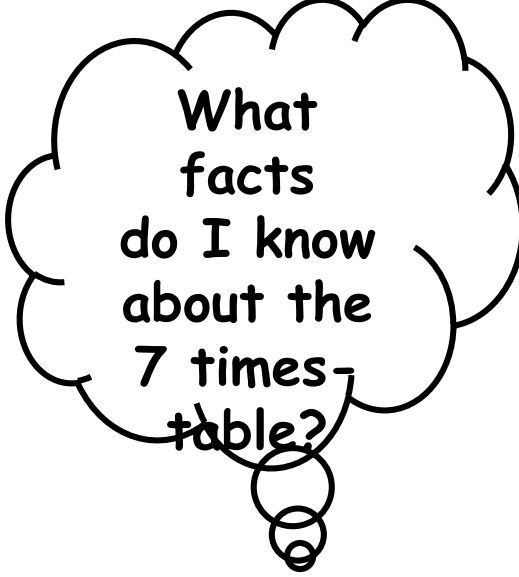
$$\begin{array}{r}
 18 \\
 - 3 \quad (1 \times 3) \\
 \hline
 15 \\
 - 3 \quad (1 \times 3) \\
 \hline
 12 \\
 - 3 \quad (1 \times 3) \\
 \hline
 9 \\
 - 3 \quad (1 \times 3) \\
 \hline
 6 \\
 - 3 \quad (1 \times 3) \\
 \hline
 3 \\
 - 3 \quad (1 \times 3) \\
 \hline
 0
 \end{array}$$

•turn horizontal number line vertical so children can see link to vertical calculation and model recording, use to illustrate need to take 'chunks' for efficiency

# Stage 4 - Chunking with a fact box (year 5 onwards)

$$100 \div 7 = \underline{14} \text{ r } 2$$

$$\begin{array}{r} 100 \\ - 70 \quad ( \underline{10} \times 7 ) \\ \hline 30 \\ - 28 \quad ( \underline{4} \times 7 ) \\ \hline 2 \end{array}$$



What facts do I know about the 7 times-table?

$$518 \div 7 = \underline{74}$$

$$\begin{array}{r} 518 \\ - 350 \quad ( \underline{50} \times 7 ) \\ \hline 168 \\ - 140 \quad ( \underline{20} \times 7 ) \\ \hline 28 \\ - 28 \quad ( \underline{4} \times 7 ) \\ \hline 0 \end{array}$$

## Fact Box

$$1 \times 7 = 7$$

$$2 \times 7 = 14$$

$$5 \times 7 = 35$$

$$10 \times 7 = 70$$

$$20 \times 7 = 140$$

$$50 \times 7 = 350$$

$$100 \times 7 = 700$$

- children need to see that when numbers are larger it is more efficient to subtract larger 'chunks'
- building a fact box will help children with the size of the 'chunks'
- children need to work with and without remainders considering if answer needs rounding up or rounding down

# Stage 5 - Short division and long division (Years 5/6)

$$96 \div 7 = 13 \text{ r } 5$$

$$7 \overline{) \begin{array}{r} 10 + 3 \\ 70 + 26 \end{array}} \text{ r } 5 \longrightarrow 7 \overline{) \begin{array}{r} 13 \\ 9 \phantom{0} \\ \phantom{0} 6 \end{array}} \text{ r } 5$$

$$\begin{array}{r} 560 \div 24 \\ 23 \text{ r } 8 \\ 24 \overline{) 560} \\ \underline{- 480} \\ 80 \\ \underline{- 72} \\ 8 \end{array}$$

- ONLY move on to these methods if understanding is secure
- show remainders as a number, fraction and decimal